Wave spectra analysis steps:

* Measure displacement, velocity, and/or acceleration in directions relative to the sensor chip.
* x and y be in the plan of the sensor and z perpendicular to the sensor, such that they form a right-hand coordinate system.
* Determine the rotation angles from the sensor’s x, y, and z, coordinate system to the glider’s coordinate system. This then gives us a rotation matrix from sensor coordinate system to the glider coordinate system.
* Rotate the sensor measurements to the glider’s coordinate system.
* From the magnetic field and/or the True North magnetometer+tilt sensors we have the pitch, roll, and heading of the glider. This gives us another rotation matrix from the glider to an earth coordinate system. Rotate the sensor’s measurements in the glider’s coordinate system to the earth’s coordinate system.
* Take the Fourier Transform of each of the sensor’s measurements in the earth’s coordinate system.
* Calculate the following spectral densities, PSD=Power Spectral Density, CSD=Cross Spectral Density:
  + PSD in the northward direction, PSD\_north, this will be real
  + PSD in the eastward direction, PSD\_east, this will be real
  + PSD in the upward direction, PSD\_up, this will be real
  + CSD in the north/east direction, CSD\_ne, this will be complex
  + CSD in the up/north direction, CSD\_un, this will be complex
  + CSD in the up/east direction, CSD\_ue, this will be complex
  + For xFFT and yFFT, the spectral density, both PSD and CSD, is:
    - fs is the sampling frequency
    - n is the number of samples in the FFT
    - CSD\_xy = (conjugate(xFFT) \* yFFT) / (fs \* n)
    - CSD\_xy[1:-1] \*= 2 to take into account negative and positive frequencies
    - CSD\_xy[-1] \*=2 if n is odd for the Nyquist frequency having positive and negative components
* Band average the PSDs and CSDs
* Calculate the five coefficients, a0, a1, b1, a2, and b2:
  + a0 is:
    - PSD\_up for displacement measurements
    - PSD\_up / (2 pi f)^2 for velocity measurements
    - PSD\_up / (2 pi f)^4 for acceleration measurements
  + a1 and b1: (N.B. no
    - denom = sqrt(PSD\_up \* (PSD\_north + PSD\_east))
    - a1 = imag(CSD\_un) / denom
    - b1 = -imag(CSD\_ue) / denom
  + a2 and b2:
    - denom = PSD\_north + PSD\_east
    - a2 = (PSD\_north – PSD\_east) / denom
    - b2 = -2 real(CSD\_ne) / denom
  + Note, a1, b1, a2, and b2 are the same for displacement, velocity, and acceleration since the normalization is applied to both the numerator and denominator, so it cancels out.
* Calculate the spectral moments:
  + df\_i is the frequency bandwidth of band i
  + m0 = sum(a0\_i \* df\_i)
  + m1 = sum(a0\_i \* f\_i \* df\_i)
  + m2 = sum(a0\_i \* f\_i^2 \* df\_i)
  + mm1 = sum(a0\_i \* df\_i / f\_i)
* Non-directional parameters:
  + Peak PSD: max(a0)
  + Hs, significant wave height: 4 \* sqrt(m0)
  + Tp, dominate period: 1/f(argmax(a0))
  + Ta, average wave period: m0 / m1
  + Tz, zero crossing period: sqrt(m0 / m2)
  + TE, mean energy period: mm1 / m0
  + Dp, dominate wave direction: arctan2(b1[argmax(a0)], a1[argmax(a0)])
* Directional parameters:
  + theta1 = arctan2(b1, a1)
  + theta2 = arctan2(b2, a2) / 2
  + dm1 = sqrt(a1^2 + b1^2)
  + dm2 = a2 \* cos(2 theta1) + b2 \* sin(2 theta1)
  + dn2 = a2 \* sin(2 theta1) + b2 \* cos(2 theta1)
  + dm1Spread = sqrt(2 (1 – dm1))
  + dm2Spread = sqrt((1 – dm2) / 2)
  + dKurtosis0 = -dn2 / dm2Spread\*\*3
  + dKurtosis1 = (6 – 8 \* dm1 + 2 \* dm2) / (2 \* (1 -dm1))\*\*2
  + r1 = sqrt(a1^2 + b1^2) / a0
  + r2 = sqrt(a2^2 + b2^2) / a0
  + D(f,theta) = (1/2 + r1 cos(theta – theta1) + r2 cos(2(theta – theta2)))/pi